1T01234 - S.E.(Information Technology Engineering)(SEM-IV)(Choice Base Credit Grading System) (R- 20-21) (C Scheme) / 41021 - Engineering Mathematics-IV

QP CODE: 10011991 DATE: 08/12/2022

(Time: 3 hours) Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

Q1 a) If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ then find the Eigen values of $A^3 + 6A^{-1} + 2I$ [5]

b) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$,along the path $(i) y = x$, $(ii)\ y = x^2$ [5]

c) Write the dual of the following problem [5]

Maximise $z = 3x_1 + 10x_2 + 2x_3$

$subject\ to$ $2x_1 + 3x_2 + 2x_3 \leq 8$

$3x_1 - 2x_2 + 4x_3 = 4$

$x_1, x_2, x_3 \geq 0$

d) A certain drug administered to 12 patients resulted in the following change in their Blood Pressure

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 [5]

Can we conclude that drug increase the Blood Pressure?

Q2 (a) Using Cauchy's residue theorem evaluate [6]

$\int_C \frac{1-2z}{z(z-1)(z-2)} dz$, Where c is |z|=1.5

(b) Verify Cayley-Hamilton theorem and find A^{-1} for $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$. Hence, find $2A^3 - A^2 - 35A - 44I$. [6]

(c) Solve by Simplex Method [8]

$Maximise$ $z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

Q3 a) Based on the following data determine if there is a relation between literacy and smoking [6]

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

(Given that Critical value of chi-square 1 d. f and 5% L.O.S is 3.841

b) Obtain Laurent's series expansion of $f(z) = \frac{1}{z^2+4z+3}$ [6]

when $(i)\ |z| < 1 \quad (ii)\ 1 < |z| < 3 \quad$ (ii) $|z| > 3$

c) Using the method of Lagrangian multipliers solve the following N.L.P.P [8]

Optimise $\quad z = {x_1}^2 + {x_2}^2 + x_3^2$

Subject to $\quad x_1 + x_2 + 3x_3 = 2$

$5x_1 + 2x_2 + x_3 = 5$

$x_1, x_2, x_3 \geq 0$

Q4a) Using the method of Lagrange's multipliers solve the following N.L.P.P [6]

Optimise $z = {x_1}^2 + {x_2}^2 + {x_3}^2 - 10x_1 - 6x_2 - 4x_3$

Subject to $\quad x_1 + x_2 + x_3 = 7$

$x_1, x_2, x_3 \geq 0$

b) Find the inverse Z-transform of $\frac{1}{z^2-3z+2}$, $if\ ROC\ is \quad (i) \quad |z| < 1 \quad (ii)\ |z| > 2$ [6]

c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. [8]

Q5a) Find $Z\{f(k) * g(k)\} \quad if \quad f(k) = \left(\frac{1}{2}\right)^k \quad , g(k) = cos\pi k$ [6]

b) Find the Eigen values and Eigen Vectors of the following matrix. [6]

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

c) Solve by the dual Simplex Method [8]

$Minimise\ z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

Q6a) Find $Z\{2^k \cos(3k+2)\}, k \geq 0.$ [6]

b) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights (i) greater than 72 inches (ii) less than 62 inches (iii) between 65 and 71 inches [6]

c) Using Kuhn Tucker conditions, solve the following NLPP [8]

$Maximise \quad z = 2{x_1}^2 - 7{x_2}^2 + 12x_1x_2$

Subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

(Time: 3 Hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1 a) If $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, then find the Eigen values of $4A^{-1} + A^3 + I$ [5]

b) Evaluate $\int_c |z|\, dz$, where C is the left half of unit circle $|z|$ =1 from z = - i to z = i. [5]

c) $Maximise \quad z = x_1 + 3x_2 + 3x_3$ [5]

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7.$

Find all the basic solutions to the above problem. Which of them are basic feasible, non-degenerate, infeasible basic and optimal solution.

d) Tests made on breaking strength of 10 pieces of a metal wire gave the following results

578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 in kgs. [5]

Test if the breaking strength of the metal wire can be assumed to be 577 kg ?

Q2 (a) Using Cauchy's residue theorem evaluate [6]

$\int_C \frac{(z+4)^2}{z^4+5z^3+6z^2} dz,$ Where c is |z|=1.

(b) Find $Z\{f(k) * g(k)\}$ $if \ \ f(k) = 4^k U(k), g(k) = 5^k U(k).$ [6]

(c) Solve the following L.P.P by Simplex Method [8]

$Maximise \quad z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Q3 a) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9: 3 :3 :1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory? [6]

(Given that Critical value of chi-square 3 d. f and 5% L.O.S is 7.81)

b) Obtain Taylor's and Laurent's series expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ [6]

c) Use the method of Lagrange's multipliers to solve the following N.L.P.P [8]

Optimize $z = 6x_1 + 8x_2 - {x_1}^2 - {x_2}^2$

Subject to $4\,x_1 + 3x_2 = 16$,

$3x_1 + 5x_2 = 15$

$x_1, x_2 \geq 0$

Q4a) fit a Poisson distribution to the following data [6]

No. of deaths	0	1	2	3	4
Frequencies	123	59	14	3	1

b) Find the inverse Z-transform of $\frac{1}{(z-2)(z-3)}$, $if\ ROC\ is\quad (i)\quad |z| < 2 \quad (ii)\ 2 < |z| < 3$ [6]

c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. [8]

Q5a) Using the method of Lagrange's multipliers to solve the following N.L.P.P [6]

Optimize $z = 4x_1 + 8x_2 - {x_1}^2 - {x_2}^2$

Subject to $x_1 + x_2 = 4,$

$x_1, x_2 \geq 0.$ [6]

b) Verify Cayley- Hamilton Theorem for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ [6]

c) Solve by the dual Simplex Method [8]

$Minimise\ \ z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3,$

$x_1 - x_2 \geq 0$, $x_1, x_2 \geq 0$

Q6a) Find the Z-transform of $f\{k\} = \begin{cases} b^k, & k < 0 \\ a^k, & k \geq 0 \end{cases}$ [6]

b) The income of a group of 10,000 persons were found to be normally distributed with mean Rs.520 and standard deviation Rs.60. Find the lowest income of the richest 500. [6]

c) Using Kuhn Tucker conditions, solve the following NLPP [8]

$Maximise\quad z = 10x_1 + 4x_2 - 2{x_1}^2 - {x_2}^2$

Subject to $2\,x_1 + x_2 - 5 \leq 0$

$x_1, x_2 \geq 0$

1T01234 - S.E.(Information Technology Engineering)(SEM-IV)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 41021 - Engineering Mathematics-IV

QP CODE: 10040268 DATE: 08/12/2023

Time: 3 hrs **Marks: 80**

Note :

1) **Q. No. 01 is compulsory.**
2) **Solve any three from Q. No. 02 to 06.**
3) **Numbers to the right indicate full marks.**
4) **Use of statistical tables is allowed.**

Q. 1. Solve.

a) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ find the sum and product of Eigen values A. 5

b) Integrate the function $f(z) = z^2$ from A(0, 0) to B(1, 1) along straight line AB. 5

c) Find the Z-Transform of $(k) = a^k$, k < 0. 5

d) A transmission channel has a per-digit error probability p = 0.01. Calculate the probability of more than 1 error in 10 received digits using Poisson distribution. 5

Q. 2.

a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. 6

b) Find the Z-Transform of Cos $\left(\frac{k\pi}{4} + \alpha\right)$ $k \geq 0$. 6

c) Use the dual simplex method to solve the LPP
Min.. $Z = 2X_1 + 2X_2 + 4X_3$
$2X_1 + 3X_2 + 5X_3 \geq 2$, $3X_1 + X_2 + 7X_3 \leq 3$, $X_1 + 4X_2 + 6X_3 \leq 5$ $X_1, X_2, X_3 \geq 0$ 8

Q. 3.

a) Evaluate $\int_C \frac{z^2}{(z-1)(z-2)} dz$ Where C is a circle $|z-1|=1$. 6

b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. 6

c) Solve the LPP by Big -M method
Maximize $Z = 3X_1 - 2X_2$
subject to $2X_1 + X_2 \leq 2$, $X_1 + 3 \geq 3$, $X_1, X_2, \geq 0$. 8

Q. 4.

a) Find inverse Z transform of $F(z) = \frac{1}{(z-1)(z-3)}$ for i) $|z| < 1$, ii) $1 < |z| < 3$. 6

b) The following data represent the marks obtained by 12 students in two tests, one held before the coaching and the other after the coaching.

Test I : 55, 60, 65, 75, 49, 25, 18, 30, 35, 51, 61, 72.

Test II : 63, 70, 70, 81, 54, 29, 21, 38, 32, 50, 70, 80. 6

Do the data indicate that the coaching was effective in improving the performance of the students?

c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z-1)(z+2)}$ about z = 0 indicating the region of convergence in each case. 8

Q. 5.

a) Determine all basic solutions to the following problem

Max. $Z = x_1 - 2x_2 + 4x_3$ 6

$x_1 + 2x_2 + 3x_3 = 7$, $3x_1 + 4x_2 + 6x_3 = 15$, $x_1, x_2, x_3 \geq 0$.

b) Using Normal distribution, find the probability of getting 55 heads in the toss of 100 fair coins. 6

c) Solve the NLPP

Optimize $Z = 10x_1 + 8x_2 + 6x_3 + 2{x_1}^2 + {x_2}^2 + 3{x_3}^2 - 100$

Subject to $x_1 + x_2 + x_3 = 20$, $x_1, x_2, x_3 \geq 0$. 8

Q. 6.

a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$. 6

b) Of the 64 off springs of a certain cross between guinea pigs 34 were red, 10 were black and 20 were white. According to the generic model these numbers should be in the ratio 9 : 3 : 4. Use 2- test to check whether the data are consistent with the model. 6

c) Max. $Z = 4x_1 + 6x_2 - {x_1}^2 - {x_2}^2 - {x_3}^2$, Subject to $x_1 + x_2 \leq 2$ and $2x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$ by K-T condition. 8

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14-May-2024 02:30 pm - 05:30 pm 1T01234 - S.E.(Information Technology Engineering)(SEM-IV)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 41021 - Engineering Mathematics-IV
QP CODE: 10054587

(Time: 3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks

Q1. (a) If matrix A = $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ find Eigen values of A^3 +5A + 8I. [5]

(b) Evaluate the integral $\int_0^{1+i} (x - y + i\,x^2)$ dz along the parabola y^2 = x. [5]

(c) Find the z-transform of f (k) = $a^k,\ k \geq 0$. [5]

(d) Maximise z = $x_1 + 3\,x_2 + 3x_3$

Subject to $x_1 + 2\,x_2 + 3x_3 = 4$

$2\ x_1 + 3x_2 + 5x_3 = 7$ find all basic solutions. Which

of them are basic feasible, And optimal basic feasible solutions. [5]

Q2 (a) Verify Cayley- Hamilton theorem for the matrix A where A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

And hence find A^{-1} *and* A^{-2}. [6]

(b) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively The sum of the squares of the deviations from the means are 26.94 and 18.73 respectively. Can The samples be considered to have been drawn from same population? [6]

(c) Solve the L.P.P by using simplex method.

Maximise z = $3\,x_1 + 2\,x_2$
Subject to $3\,x_1 + 2\,x_2 \leq 18$;
$0 \leq x_1 \leq 4$;
$0 \leq x_2 \leq 6$;
$x_1, x_2 \geq 0$. [8]

Q3 (a) Find the Laurent's series for

F (z) = $\frac{4z+3}{(z-3)\ z\,(z+2)}$ valid for $2 < |z| < 3$. [6]

(b) Using the method of Lagrange's multiplier solve the N.L.P.

Optimise $z = 12\,x_1 + 8\,x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23.$

Subject to $x_1 + x_2 + x_3 = 10.$ $x_1, x_2, x_3 \geq 0.$ [6]

(c) Marks obtained by students in an examination follow normal distribution. If 30 % Of the students got below 35 marks and 10 % got above 60 marks. Find the mean and standard deviation. [8]

Q4 (a) Find the Eigen values and Eigen vectors of matrix A = $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ [6]

(b) Find inverse z- transform of F (z) = $\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$ $3 < |z| < 4.$ [6]

(c) Using the Kuhn –Tucker conditions solve the N.L.P.P [8]

Maximise $z = 12\,x_1x_2 + 2\,x_1^2 - 7\,x_2^2$

Subject to $2\,x_1 + 5x_2 \leq 98\,;$

$x_1, x_2 \geq 0.$

Q5 (a) Show that the matrix A = $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and the Diagonal zing matrix M. [6]

(b) Find the relative maximum or minimum of the function

$z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100.$ [6]

(c) Evaluate $\oint \frac{2z-1}{(2z+1)\,z\,(z+2)}$ dz using Cauchy's residue theorem, where C is the circle $|z| = 1.$ [8]

Q6 (a) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use χ^2- test To check whether these frequencies are in agreement with that occurrence was The same during 10 months period. Test at 5 % level of significance. [6]

(b) Find z – transform of [2^k cos (3k + 2)] , k ≥ 0 . [6]

(c) Use the dual simplex method to solve the L.P.P. [8]

Minimise z = $2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$;

$4x_1 + 3x_2 \geq 6$;

$x_1 + 2x_2 \leq 3$;

$x_1, x_2 \geq 0$.

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